Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 2 9 1 2 4 5 7 0 3 6 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC21 06_9709_33/RP

1 Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

2 Solve the equation $4^x = 3 + 4^{-x}$. Give your answer correct to 3 decimal places. [5]

3 The parametric equations of a curve are

$$x = t + \ln(t + 2), \qquad y = (t - 1)e^{-2t},$$

where $t > -2$.

(a) Express $\dfrac{dy}{dx}$ in terms of t, simplifying your answer. [5]

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(b) Find the exact y-coordinate of the stationary point of the curve. [2]

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4 Let $f(x) = \dfrac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions. [3]

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(b) Hence find $\int_1^2 f(x)\,dx$, giving your answer in the form $\ln\left(\dfrac{a}{b}\right)$, where a and b are integers. [4]

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5 **(a)** By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2}\tan\theta$ may be expressed as $\tan^4\theta + 2\tan^2\theta - 7 = 0$. [4]

(b) Hence solve the equation $\tan 4\theta = \frac{1}{2}\tan\theta$, for $0° < \theta < 180°$. [3]

6 **(a)** By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leqslant \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 1.5. [2]

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(c) Use the iterative formula $x_{n+1} = 2\tan^{-1}\left(\dfrac{1}{1+e^{-x_n}}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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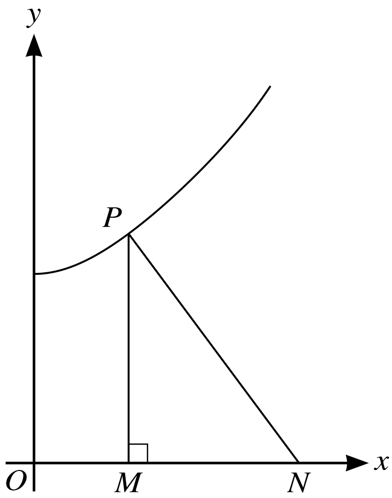

For the curve shown in the diagram, the normal to the curve at the point P with coordinates (x, y) meets the x-axis at N. The point M is the foot of the perpendicular from P to the x-axis.

The curve is such that for all values of x in the interval $0 \leqslant x < \frac{1}{2}\pi$, the area of triangle PMN is equal to $\tan x$.

(a) **(i)** Show that $\frac{MN}{y} = \frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

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(ii) Hence show that x and y satisfy the differential equation $\frac{1}{2}y^2\frac{\mathrm{d}y}{\mathrm{d}x} = \tan x$. [2]

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(b) Given that $y = 1$ when $x = 0$, solve this differential equation to find the equation of the curve, expressing y in terms of x. [6]

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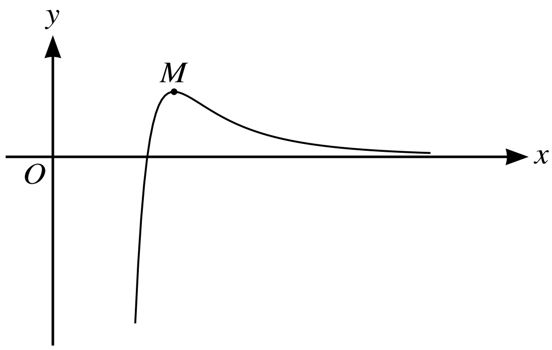

The diagram shows the curve $y = \dfrac{\ln x}{x^4}$ and its maximum point M.

(a) Find the exact coordinates of M. [4]

(b) By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4}\, dx < \frac{1}{9}$. [6]

9 The quadrilateral $ABCD$ is a trapezium in which AB and DC are parallel. With respect to the origin O, the position vectors of A, B and C are given by $\overrightarrow{OA} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

(a) Given that $\overrightarrow{DC} = 3\overrightarrow{AB}$, find the position vector of D. [3]

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(b) State a vector equation for the line through A and B. [1]

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(c) Find the distance between the parallel sides and hence find the area of the trapezium. [5]

10 **(a)** Verify that $-1 + \sqrt{2}\mathrm{i}$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

(b) Find the other roots of this equation. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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